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Amplified spontaneous emission
III. Intensity and saturation

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Abstract. A theory is developed to account for the intensity of ASE as a function of the length and diameter of the amplifying medium and of its inversion density. Detailed results are presented from a study of the 3·39 μm transition in He–Ne which verify the theory. The theory is also shown to fit the experimental data obtained from the 0·337 μm transition in N_2.

1. Introduction

In paper I of this series (Peters and Allen 1971 to be referred to as I) we derived and experimentally confirmed the threshold condition for the onset of amplified spontaneous emission (ASE). In paper II (Allen and Peters 1971 to be referred to as II) a relationship was derived between this condition and the threshold condition for laser action to occur on the same atomic transition; again the relationship was confirmed experimentally.

In this paper we develop a theory to account for the way in which the output intensity of ASE varies as a function of the length, diameter and inversion density of the amplifying medium. Careful note is taken of the fact that for column lengths greater than the critical length for the onset of ASE, L_c, not only will there be effects due to stimulated emission but spontaneous emission will still occur depopulating the upper level. A fraction of this spontaneous emission will contribute to the output intensity along with that emission induced by the field. Finally, although the approach is a rate-equation one, the equations are solved in full for the steady state and the effects of saturation are dealt with properly. Saturation occurs because the inversion density is modified by the two travelling ASE waves as they grow and pass through the amplifying medium.

The second half of the paper gives an account of the experimental confirmation of this theory using the 3·39 μm system of He–Ne. The theory is also applied to the results obtained by Leonard (1965) in his investigation of the N_2 system at 0·337 μm, and found to fit with an encouraging degree of success.

No comparable theory exists in the literature. Yariv and Gordon (1963) considered amplification due to an inverted population of atoms but did not allow for saturation or for any of the effects of spontaneous emission. Kogelnik and Yariv (1964) effectively considered the power output of ASE by analysing the effect of noise in a laser amplifier. They considered the power of spontaneous emission emitted by a volume of excited gas into a solid angle $d\Omega$ in the range $\nu \rightarrow \nu + d\nu$ and its subsequent amplification, but again saturation was not taken into account.

Although the 3·39 μm system is well known to have a high gain and to operate without mirrors (Rigden and White 1963), the documented information about its ASE behaviour is again very limited. Rigden and White estimated that the system had a gain of up to about 50 dB m^{-1} for a tube 100 cm long but did not investigate the gain as a function of the parameters involved. Andronova *et al.* (1968) investigated the output of a 3·39 μm system as a function of gain and found in regions of low saturation that the theory of Kogelnik and Yariv gave quite good results to within a factor of 2–2·5 for the power output. But when high saturation occurs the computed and experimental values of the output power are at variance by an order of magnitude. These discrepancies arise because their output power measurements are associated with a measurement of beam divergence which is itself an order of magnitude too large. Their technique for measuring the divergence and their interpretation of the values obtained seem to be of arguable validity. The actual physical length and diameter of the tube was, in any case, not varied.

2. Theory

Consider a column of atoms which has a population inversion density at position x and time t between a particular pair of energy levels 2 and 1 of (n_2-n_1). Let the cross-sectional area of the column be a and its length L. Both levels are steadily populated at rates R_2 and R_1 respectively from a source level assumed to have a population n_0, and both lose population by radiative decay to other levels with effective probabilities A_2 and A_1. Level 2 also loses population to level 1, thus helping to destroy the population inversion, with probability A. Simple rate equation may be written for the time dependence of the population densities at position x as

$$\frac{\partial n_2}{\partial t} = -(n_2-n_1)\frac{\sigma}{a}(N(x)+N(L-x))-An_2-A_2n_2+R_2n_0$$

$$\frac{\partial n_1}{\partial t} = (n_2-n_1)\frac{\sigma}{a}(N(x)+N(L-x))+An_2+R_1n_0-A_1n_1$$

where $N(x)$ is the number of photons threading the cross-section a per second at position x and travelling in the positive x direction, and $N(L-x)$ is the number at the same position travelling in the opposite direction. σ is the resonance absorption cross-section for the transition $1 \rightarrow 2$. Assuming steady state conditions, that is $\dot{n}_2 = \dot{n}_1 = 0$, yields for the upper level population density

$$n_2 = \frac{B+C(N(x)+N(L-x))}{1+E(N(x)+N(L-x))}$$

and for the inversion density

$$n_2-n_1 = \frac{F}{1+E(N(x)+N(L-x))}$$

at position x in the tube.

The equations are immediately simplified if $N(x)+N(L-x)$ is assumed constant for all x which implies of course that the population densities are independent of x. If this assumption is made then,

$$N(x)+N(L-x) = G$$

and the above relationships reduce simply to,

$$n_2 = \frac{B+CG}{1+EG}$$

and

$$n_2-n_1 = \frac{F}{1+EG}.$$

That the assumption is justifiable in the 3·39 μm He–Ne system will be demonstrated in the experimental section of this paper.

The rate of increase in N for one of the travelling waves with distance, in the steady state, is given by

$$\frac{\partial N}{\partial x} = \{(n_2 - n_1)\sigma - \nu a\}N + \frac{An_2 a\Delta\Omega}{4\Pi}$$

where the population densities are now independent of x; $\Delta\Omega$ is the solid angle into which spontaneously emitted photons may be emitted and so contribute to the output power and is, of course, a function of x, and ν is a loss term which accounts for all mechanisms by which photons are lost from the final output beam in the x direction.

Thus, substituting for the inversion densities previously found,

$$\frac{\partial N}{\partial x} = -X(L)N + Y(L)\frac{\Delta\Omega}{4\Pi} \tag{1}$$

where

$$X(L) = -\left\{\left(\frac{F}{1+EG}\right)\sigma - \nu a\right\}$$

$$Y(L) = \left(\frac{B+CG}{1+EG}\right)Aa.$$

It may be immediately recognized that $-X(L)$ is equivalent to what is usually called the gain coefficient of the medium. Reorganizing the form of $X(L)$ gives,

$$X(L) = \frac{K_1'G - K_2}{1+K_3'G}$$

where K_1', K_2 and K_3' are all positive. K_2 may be recognized as the small signal gain coefficient, K_3' represents the effect of saturation—see the approach of Rigrod (1963) for a similar result with respect to saturation—and K_1' is the loss constant. In a gas $K_1' \sim 0$, but in a dye or solid scattering losses could be significant and K_1' could be important.

Equation 1 may be integrated to give

$$[N\exp(X(L)x)]_{L_c}^{L} = Y(L)\int_0^{L-L_c}\frac{\Delta\Omega}{4\Pi}\exp(X(L)x)\,\mathrm{d}x.$$

The limits of the two resulting integrals are of some interest. The left hand side represents stimulated emission at all lengths from L_c to L. The right hand side is concerned only with that part of the tube for which spontaneous emission can act as a source term for stimulated emission. For lengths in excess of $(L-L_c)$ the spontaneous emission will provide noise, but will not be able to grow and become a source of stimulated emission. The absence of stimulated emission at lengths below L_c has the strict logical implication, if the assumption $N(x)+N(L-x) = G$ is to be believed, that there is no amplification of the ASE when $x > L-L_c$. This is clearly not the case; however, the above equation does take account of the amplification in this range and the assumption, although obviously an approximation, is shown to be justified in the next section. Since at L_c the photon number $N(L_c)$ is effectively zero, the equation further simplifies to,

$$N(L)\exp(X(L)L) = Y(L)\int_0^{L-L_c} \frac{\Delta\Omega}{4\Pi}\exp(X(L)x)\,dx \qquad (2)$$

and for this to be solved an explicit form for $\Delta\Omega$ is required.

It can be shown that the solid angle subtended by the end aperture of the tube at a point $(L-x)$ from the end and a distance w off-axis is, for a tube of radius d,

$$\Delta\Omega = 2\Pi\left[1-\left[\tfrac{1}{2}+\tfrac{1}{4}\left\{\left(\frac{(L-x)^2+(d+w)^2}{(L-x)^2+(d-w)^2}\right)^{1/2} + \left(\frac{(L-x)^2+(d-w)^2}{(L-x)^2+(d+w)^2}\right)^{1/2} - \frac{4d^2}{\{(L-x)^2+(d-w)^2\}^{1/2}\{(L-x)^2+(d+w)^2\}^{1/2}}\right\}\right]^{1/2}\right].$$

For a tube with a length appreciably greater than the tube radius the variation over the tube cross section may be ignored and

$$\Delta\Omega = 2\Pi\left(1-\frac{(L-x)}{\{(L-x)^2+d^2\}^{1/2}}\right).$$

However, given the integral in equation (2), the smallest value of $(L-x)$ involved is L_c and it is sufficient to expand $\Delta\Omega$ binomially and take the first term $\pi d^2/(L-x)^2$. For a high gain system as a dye where d and L may well be of the same order of magnitude this approximation would be extremely unreasonable.

Substituting this form of $\Delta\Omega$ into (2) and changing the variable of integration to $y = (L-x)$ gives

$$N(L) = \frac{Y(L)d^2}{4}\int_{L_c}^{L} \frac{\exp(-X(L)y)\,dy}{y^2}. \qquad (3)$$

This can then be integrated by parts to get just a y-dependence in the denominator of the integrand. This integral can then be expressed as an integral function which is tabulated as a function of the exponent $X(L)$ and the limits of integration. This, however, leaves a cumbersome expression to be fitted to the experimental values, and so the integration above was performed numerically with a computer.

If the assumption is made that the dominant variable under the integral is the exponential numerator and that y^2 varies very much less quickly, which implies that an average value of $\Delta\Omega$ is being taken, then the integral can be solved analytically to give

$$N(L) = \frac{Y(L)d^2}{X(L)(L+L_c)^2}(\exp(-X(L)L_c) - \exp(-X(L)L)).$$

Although this expression is clearly not of general validity, its implicit assumption only holds near threshold, it nevertheless proves useful as a means of acquiring some information about the magnitudes of the coefficients K_2 and K_3' in equation (3) when an attempt is made to fit the theory to the results of experimental measurement.

3. Comparison of theory with experiment

The experimental arrangement for the 3·39 μm He–Ne system was the same as that described in paper I. The He–Ne was rf-excited and a range of tubes varying from 2 to 4 mm bore were investigated. In this part of the experiment, to enable a greater range of intensity against length to be investigated, a silvered mirror was placed at the end of the tube furthest from the detector although as near to the Brewster angle window as was possible. Measurements in the range L_c to 310 cm were taken in the absence of the mirror and measurements at greater lengths up to 620 cm were taken with the mirror in place. Clearly the finite mirror transmission coefficient and the divergence of the beam ensured that this system was more lossy than a simple 620 cm long tube, but a comparison with a tube length of 310 cm in the absence of a mirror with that of length 155 cm with a mirror, showed that there was agreement in the output intensities to less than 5%.

It is interesting to see what effect the photon flux, described by G, has on the spontaneous emission, n_2. It is not sufficient to say that the spontaneous emission at 0·633 μm was investigated to determine the x-dependence of n_2 and found to be constant over the whole tube length, because in paper I it was stated that the discharge was evenly maintained by monitoring the spontaneous emission at 0·633 μm, and consequently the non-dependence of n_2 on x would be expected. It seems worthwhile to describe in a little more detail the procedure adopted for monitoring the spontaneous emission at 0·633 μm. Firstly, the three-metre-long column of gas was excited in three independent, equal sections by three rf oscillators. So that if only one section was excited at any particular instant there was no stimulated emission ($L < L_c$), and $n_2 = B$. The discharge could then be investigated for evenness by monitoring the spontaneous emission at 0·633 μm at different points along that section. Each section was prepared separately to give a constant value of n_2. So in the absence of stimulated emission there was then constant spontaneous emission at 0·633 μm at *all* points along the tube. When all three sections were excited together, stimulated emission occurred along the tube, and whether or not n_2 was a function of x could be determined by re-monitoring the spontaneous emission at different points along the tube. It was found that the spontaneous emission did not depend upon x to within the accuracy with which the discharge could be evenly maintained in each section ($\sim$ 5%), although this does not necessarily imply 5% constancy in G.

It is now necessary to obtain a value for the constant, G. In the range $L_c < x < (L-L_c)$ the total photon number is $N(L-L_c)$ and in the range $0 < x < L_c$ and $(L-L_c) < x < L$ the number varies between $N(L-L_c)$ and $N(L)$. The best value for G would thus seem to be the average value of the total photon number, $\bar{N}_L$. If, for simplicity the constant used was $N(L)$ rather than $\bar{N}_L$ the saturation term would be made larger than it ought to be. Such an approximation would introduce a potentially large percentage error in this term near threshold, but near threshold the saturation term is very small and the final error introduced is in consequence negligible. At high intensity when saturation is strong the difference between $\bar{N}_L$ and $N(L)$ is small and would introduce a negligible error into the denominator of the term $X(L)$. It is in the intermediate region above threshold but below the limit of strong saturation that the error introduced will have the greatest effect. Consequently since this is the regime into which the 3·39 μm transition and the 0·337 μm N_2 transition fit it was necessary to put G equal to $\bar{N}_L$.

The output intensity was measured as a function of length for four different inversion densities and four tube bores. For a particular inversion and bore the intensity against length results were fitted by computer to

$$I(L) = K\int_{L_c}^{L} \frac{\exp(-X(L)y)\,dy}{y^2}$$

where

$$X(L) = \frac{-K_2}{1+K_3\bar{I}_L}.$$

It will be noted that the spontaneous emission term $Y(L)$ has been incorporated into the constant term K, even though L is a variable. This is completely justifiable for the He–Ne system, the justification being very similar to that previously described for testing the x-dependence of n_2. The discharge was maintained evenly as before. The spontaneous emission at a point in one section was monitored, then the second and finally the third section was excited, with the observation point remaining the same. The variation of all three readings was less than $\frac{1}{4}\%$. Certainly for the tube lengths employed in this experiment the approximation is a justifiable one.

The fit for the highest inversion used, shown as 15·8 units on the graph and corresponding to a gain coefficient of $1{\cdot}58\times10^{-2}$ cm^{-1}, and a tube bore of 2·5 mm, yielded the values $K = 4{\cdot}02\times10^2$, $K_2 = 1{\cdot}75\times10^{-2}$ and $K_3 = 5{\cdot}79\times10^{-4}$. The numerical value of K is not of any intrinsic interest since it includes a scaling factor which translates a photon number into a photoelectric current, accounting for the optics of the light collecting device. It might be noted that K_2 compares well with the measured value of the gain coefficient. The results of the above fit are plotted in figures 1 and 2, where the points are experimental results and the smooth curve the

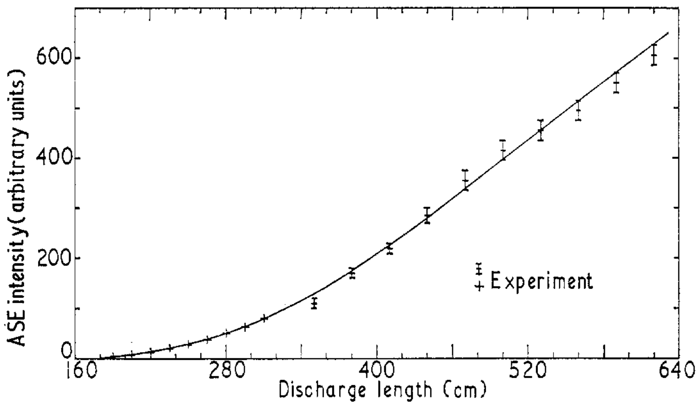

Figure 1. Experimental points and theoretical curve (full line) for the variation of intensity with length for the 3·39 μm, He–Ne ASE transition. ($n_2 - n_1$ = 15·8 units; bore = 0·25 cm).

theoretical fit. The constants obtained allow the intensity I to be predicted as a function of tube bore for constant inversion density and tube length. Figure 3 gives a typical comparison of theory with experiment. The tube bore occurs in terms K and K_3 but at small tube lengths when $X(L) \sim -K_2$ the dependence only occurs in K. Similarly the variation of I as a function of inversion was predicted for constant length and bore. The results shown in figure 1 of paper I were used and the comparison of theory and experiment is shown in figure 4. The inversion density occurs in the terms K and K_2. In K it is strictly the upper level population density but, as already discussed (in I), this is to a very good approximation the inversion density.

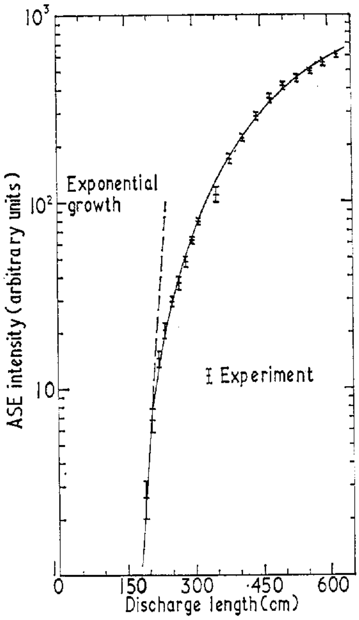

Figure 2. Experimental points and theoretical curve (full line) for the variation of lg intensity) with length, showing the region of exponential growth, for the 3·39 μm, He–Ne ASE transition ($n_2 - n_1$ = 15·8 units; bore = 0·25 cm).

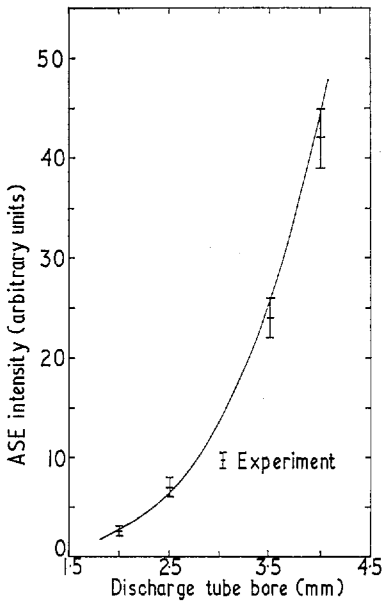

Figure 3. Experimental points and theoretical curve (full line) for the variation of intensity as a function of tube bore for an inversion density of 15·8 units and a tube of length 205 cm, for the 3·39 μm, He–Ne ASE transition.

In I of this series some results were quoted concerning ASE in the 0·614 μm system in Ne. Unfortunately this system is unsuitable for the purposes of this paper. The initial exponential growth occurs over a very short range of tube lengths (of the order of 1 cm) and the construction of Pyrex tubes to an accuracy in length of greater than $\pm\frac{1}{2}$ cm is formidable. Further the measurements are hindered by possible 'pulse chasing' effects in longitudinal pulse excited systems, where the exciting current pulse may be 'overtaken' by the ASE light pulse along the tube before it reaches the earthed electrode. This would manifest itself as a levelling off of the ASE intensity as the length of the tube is increased above a certain value (this is possibly what is happening in figure 3 of I). However, Leonard (1965) has experimentally studied the build-up of ASE intensity with length for the N_2 second positive system at 0·337 μm. He used a cross-field excitation system where pulse chasing effects are absent due to the small dimension of the nitrogen discharge tube in the direction of the electric field and where the length of the excited gas is easily and accurately varied by disconnecting electrodes. The one disadvantage of the nitrogen system is that the ASE output is not caused by a simple electronic transition as in neon. The resulting output is due to several rotational lines associated with the (0, 0) band of an electronic transition. So, in the absence of a very high resolution monochromator ($\lambda/\Delta\lambda \gtrsim 10^{+5}$) an average intensity over several ASE lines is observed.

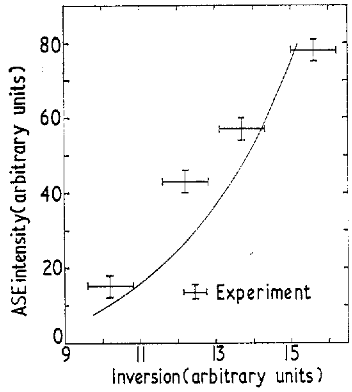

Figure 4. Experimental points and theoretical curve (full line) for the variation of intensity as a function of inversion density for a tube bore of 0·25 cm and length 310 cm, for the 3·39 μm He–Ne ASE transition.

In consequence it is not surprising that the quality of the fit between experiment and theory is not perfect. Figures 5 and 6 compare theory and experiment and it should be realized that both I and L have been estimated from Leonard's published graph. Furthermore L_c was predicted via the threshold condition for ASE (see paper I) and the theory assumes that a steady-state exists. A conservative 5% error bar has been put on each point in Leonard's results to show the degree of agreement between theory and experiment, the quality of which is extremely encouraging.

One final remark on the fitting of this theory to Leonard's results should be made concerning the term $Y(L)$ which has again been taken as being independent of L. For the He–Ne system this was justified, but may not be for that of Leonard. Note that the horizontal axes in figures 5 and 6 have been multiplied by a factor two as

Leonard plotted 'actual' discharge length, but the mirror effectively acts to double this. This results in the small signal gain coefficient he deduces of $1{\cdot}73 \times 10^{-1}$ per cm (equivalent to a gain of 3×10^7/m) being a factor of two too large. Hence this coefficient ought really to be $8{\cdot}65 \times 10^{-2}$, this compares with our computer fitted result for the full theory of $K_2 = 9{\cdot}78 \times 10^{-2}$, where $K_3 = 6.95 \times 10^{-6}$ and $K = 3{\cdot}42$.

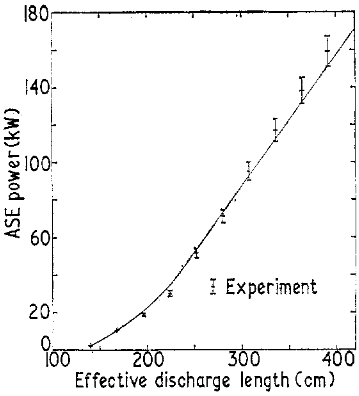

Figure 5. Experimental points and theoretical curve (full line) for the variation of intensity with length for the 0·337 μm, N_2 ASE transition.

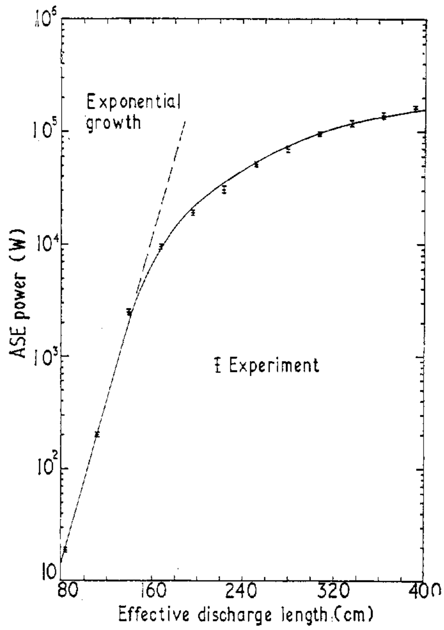

Figure 6. Experimental points and theoretical curve (full line) for the variation of lg (intensity) with length, showing the region of exponential growth, for the 0·337 μm, N_2 ASE transition.

The range of output powers embraced by these ASE systems is very large. The output of the He–Ne system is of the order of a milliwatt, and that of the N_2 is of the order of a few hundred kilowatts. It seems very satisfactory that the theory should fit so well in each case.

4. Conclusions

Although a rate equation was involved, and the solutions have been obtained in the limit of several approximations, the theory appears to be successful. In earlier work (Allen and Peters 1970) and in I and II some discussion has been given of the conflict between the idea of coherent interactions, and the more simply described concept of stimulated emission. Again some awareness of this conflict seems necessary.

For example, in the nitrogen system for all lengths greater than L_c, there is a pulse travelling through an amplifying medium. The degree of inversion is high and with a pulse duration of the order of 20 ns and an upper level lifetime of about 40 ns, conditions seem at least partially suitable for the propagation of a π pulse, (see Icsevgi and Lamb 1969, Hopf and Scully 1969). Indeed Shipman (1967) has produced 4 ns pulses in N_2 by preparing the inversion sequentially along the length of plasma so that the pulse always found a maximum inversion. Yet, as in Leonard's work, there seems to be no evidence of pulse decomposition or ringing.

Possibly, experimental work of a higher accuracy will demonstrate coherent effects, but to the level of accuracy employed so far, it does appear that the rate equation approach is sufficient. It is not surprising that the He–Ne system does not lend itself to coherent effects. The nature of the continuous excitation will destroy the phase relationships between dipole moments that must be achieved for collective effects to be seen.

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